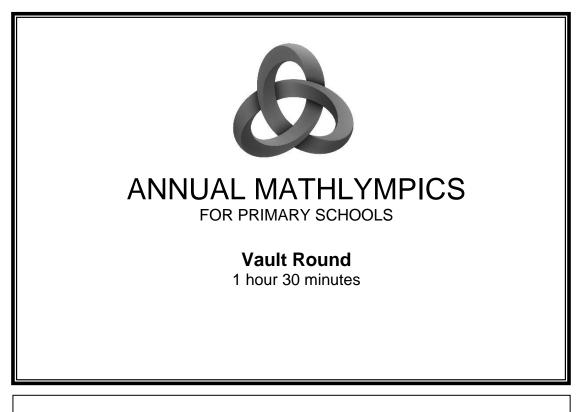
# NAME OF MATHLYMPIAN: \_\_\_\_\_



Instructions to Mathlympians

- 1. Do not open the booklet until you are told to do so.
- 2. Attempt ALL 28 questions.
- 3. Diagrams are not drawn to scale.
- 4. Write your answers neatly on the ANSWER SHEET provided.
- 5. Marks are awarded for correct answers only.
- 6. No calculators may be used.

Questions in Section A carry 2 marks each, questions in Section B carry 4 marks each and questions in Section C carry 5 marks each.

This paper consists of Questions 1 to 28 on pages 1 to 10.

## Section A

Each of the questions 1 to 10 carries 2 marks.

1. Abel's age in 2019 will be equal to the sum of the digits of his year of birth. If Abel is an adult, how old is Abel now?

2. Given that  $6^2 = 6 \times 6$ ;  $6^3 = 6 \times 6 \times 6$ ;  $6^4 = 6 \times 6 \times 6 \times 6$ ; etc, what is the sum of the last two digits of  $6^{2018}$ ?

3. Find the sum of all the numbers less than 1000 that are divisible by both 5 and 7.

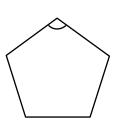
4. Three friends, Caria, Dennis and Emma are out shopping for packets of candies. As Caria did not bring any money, Dennis and Emma split the bill equally. Caria took half of all the packets, Dennis took a third of the remaining packets and Emma took the rest. The next day, Caria came with \$15 to return to her friends. How much should she return to Emma?

5. Fifty students with index numbers 1 to 50 are waiting in a room. A teacher comes in and calls out the first 3 prime numbers in sequence. Students with index numbers corresponding to the called-out prime numbers and their multiples, left the room. How many students remained in the room?

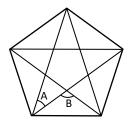
6. A number is a palindrome if it is the same backwards as forwards. An example of a palindrome is 12321. Find the largest palindrome that can be formed from the product of two 2-digit numbers.

7. The first five terms of a sequence is 52 45 38 31 24 . Write an expression to find the *n*th term of this sequence in the format n = n.

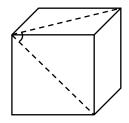
The figure shown is a polygon known as a pentagon.
 Because each of the sides are equal, we call it a regular pentagon.
 Find one of the interior angles of a regular pentagon.



9. The figure shows a star drawn inside a regular pentagon. Find the difference of  $\angle A$  and  $\angle B$ .



10. What is the angle inside the cube that is between the 2 diagonals as shown below?



<u>Section B</u> Each of the questions 11 to 20 carries 4 marks.

11. Solve 
$$\frac{4 \times A + \frac{1}{2} + 3 \times 2018 + 2A \times 22}{4 \times A + \frac{1}{6} + 2 \times 1009 + A \times 12} \times 3$$

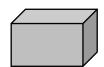
1ABCDE  $\times$ 3 12. The product of a 6-digit number, 1ABCDE and 3, is ABCDE1, that is, ABCDE1 What is the 5-digit number ABCDE?

13. How many 3-digit numbers which are multiples of 7 end with the digit 4?

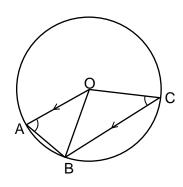
14. When two prime numbers are separated by a single number, they are known as prime pairs. An example of a prime pair is 29 and 31. What is the highest common factor of all the numbers found between each 3-digit prime pairs?

15. Jan's father is thrice the age of Jan. In *n* years' time, Jan's father will be twice Jan's age then. How many times is Jan's father's age in *n* years compared to the age of Jan now?

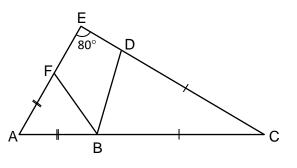
16. The volume of a solid cuboid is 144 cm<sup>3</sup>. The perimeter of the top face of the cuboid is 22 cm. If each side of the cuboid is a whole number of centimetres and longer than 2 cm, find the height of the cuboid.



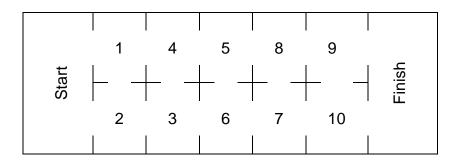
17. In the diagram, O is the centre of the circle and A, B and C are points on the circumference.
OA is parallel to CB. The ratio of ∠OAB and ∠OCB is 5 : 2. Find ∠AOC.



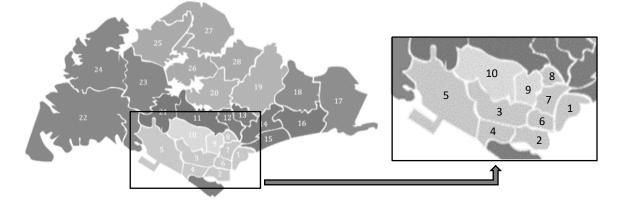
18. Given that AB = AF and BC = CD and  $\angle$ DEF = 80°. Find  $\angle$ DBF.



19. The diagram below shows the layout of 10 connected rooms. How many ways are there to get from the Start corridor to the Finish corridor if you can only move from a room of a smaller number to one with a larger number?



20. Below is Singapore's 28 District Code map.



At most how many ways are there of colouring the central district (No. 1 - 10) using three different colours such that adjacent districts do not share the same colour?

<u>Section C</u> Each of the questions 21 to 28 carries 5 marks.

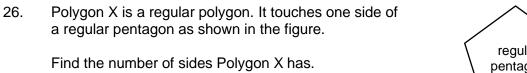
21. On the first day, Elisa read  $\frac{1}{6}$  of a story book. The next day, she finished another 40 pages. On the third day, what she read was 1.4 times as many pages as the first 2 days combined. She finished the last  $\frac{1}{3}$  of the book on the fourth day. How many pages are there in the book?

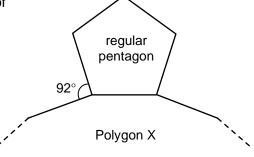
22. Amy planned to cover the 8 m by 8 m floor of her living room with a square carpet in the middle and with wooden tiles along the borders. The carpet and wooden tiles cost \$40/m<sup>2</sup> and \$15/m<sup>2</sup> respectively and she paid \$1360 in total. What is the width of the wooden tiled border?

23. If we write out the digits of all the positive integers into a sequence of digits starting from 1, that is, 1 2 3 4 5 6 7 8 9 1 0 1 1 1 2 ..., we will encounter the first of three consecutive 5s in the 100<sup>th</sup> position of the sequence. At which position will we encounter the first of five consecutive 5s?

24. Find how many consecutive zeros there are at the end of the product of the following multiplication: 166 × 172 × 178 × ... × 598.

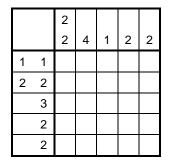
25.  $\ln \frac{1}{A + \frac{1}{B + \frac{1}{C}}} = \frac{7}{10}$ , each letter represents a distinct digit. What are the values of A, B and C?

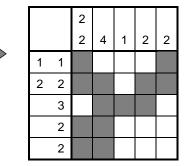




27. A 200-m long train travelling at the speed of 30 km/h closed up on a jogger at 7:45 a.m. It took 30 seconds for the train to completely pass the jogger. At 7:54 a.m., the train came upon a cyclist. It took the train 15 seconds to completely pass the cyclist. At what time will the cyclist and the jogger meet?

28. Nonograms are logic puzzles where the numbers along the top indicate the sequences and numbers of consecutive cells down the respective columns which have to be shaded; and the numbers along the left indicate the sequences and numbers of consecutive cells along the respective rows which have to be shaded. A sample nonogram and its solution is shown below.

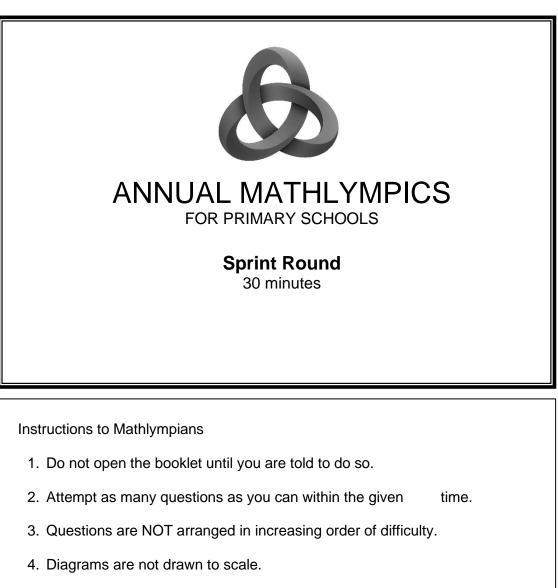




Shade the nonogram given in your Answer Sheet.

				1		1							
				1		1	2					4	2
				2	3	1	1	1	4	4		1	2
				3	4	2	2	7	2	1	3	1	2
1	1	1	1										
1	1	1	1										
	4	1	1										
	1	2	1										
1	1	3	1										
	2	2	4										
1	1	2	1										
	2	2	3										
		6	1										
1	1	3	2										

## NAME OF MATHLYMPIAN: \_\_\_\_\_



- 5. Write ALL your answers neatly on the answer sheet provided.
- 6. Marks are awarded for correct answers only.
- 7. You may use the Casio calculator provided.

All questions carry 1 mark each.

### Sprint Section

Each of the questions 1 to 30 carries 1 mark.

- 1. My age is 45 divided by  $\frac{1}{5}$  of my age. How old am I?
- 2. If you multiply all the odd numbers between 1 and 2018, what is the last digit of the product?
- 3. A student scored an average of 23 marks for several weekly tests. After the final test, his average marks became 22.25. What did he score on his final test if his score on every test was between 18 and 30 marks?
- 4. From all the 4-digit numbers which can be formed using the digits 3, 4, 6, and 7 without repetition, how many are exactly divisible by 44?
- 5. What is the smallest 3-digit square number which is also the difference of two other square numbers?
- 6. What is the largest possible number of prime numbers that can occur in a set of 20 consecutive numbers greater than 50?
- 7. An animal trainer has 5 tigers and 4 lions. He would like to line them up so that no two lions and no two tigers are together. In how many ways can he line up his animals?
- 8. The ratio of the lengths of the two diagonals of rhombus ABCD is 3 : 4. The sum of their lengths is 56 cm. What is the perimeter of rhombus ABCD?
- 9. A cube container can hold 1728 litres of water. How many cube boxes of sides 25 cm can fit inside the cube container?
- 10. In the equation  $\frac{(x+1)}{1} + \frac{(x+2)}{2} + \dots + \frac{(x+100)}{100} = 100$ , what is x?
- 11. A couple has 7 children. Each child is born exactly 18 months apart. If the sum of the ages of the three youngest children is 29 years, what is the sum of the ages of the three oldest children?
- 12. A total of 60 blackbirds were sitting in three trees. Suddenly, 6 blackbirds flew away from the first tree, 8 from the second, and 4 from the third. After that, there were twice as many birds in the second tree as the first, and twice as many in the third tree as the second. How many birds were originally in the second tree?

- 13. A group of men and women went shopping. Each man in the group spent \$200, each woman spent \$160, and the total they spent was \$2640. If there were an odd number of people in the group, how many of them were men?
- 14. Norman wrote down nine numbers in increasing order. The middle number is the average of all nine numbers. The average of the first five numbers is 27 and the average of the last five numbers is 49. What is the sum of all the numbers?
- 15. The sum of the digits in a 4-digit number is 9. None of the digits is 0. The 4-digit number is a multiple of 5 greater than 1905. What is the digit in the Tens' place?
- 16. A number is called stuttering if all its digits are 1. How many whole numbers are there, between 1 and 100 000, which can be multiplied by 33 to give a stuttering product?
- 17. A large box contains thousands of marbles of 20 different colours all mixed together. The marbles are sold only by weight as they are being scooped out randomly. Each marble weighs 5g. What is the weight of all the marbles you must buy to ensure that you get at least 100 marbles of the same colour? (Give your answer in g.)
- 18. If 97 336 cubes were put together to form a huge cube, how many of these cubes cannot be seen from the outside?
- 19. If  $\frac{2x-y}{x+y} = \frac{2}{3}$ , what is  $\frac{x}{y}$ ?
- 20. The numbers represented by a, b, c, d, and e are greater than 0 and satisfy the conditions ab = 2, bc = 3, cd = 4, and de = 5. What is  $\frac{e}{a}$ ?
- 21. Two litres of fruit juice containing 10% sugar are mixed with 3 litres of another juice containing 15% sugar. What percentage of the mixture now consists of sugar?
- 22. For every pot of roses that Jimmy delivered to a nursery, he was paid \$1.20. But if he damaged any pot, he would not be paid for that pot's delivery and he had to pay \$2.00 as penalty. If he was paid a total of \$568 to deliver 500 pots of roses, how many pots of roses did he damage?
- 23. Cathy and Roy had some candies. If Cathy and Roy ate 8 candies and 4 candies respectively each day, Cathy would still have 50 candies when Roy had eaten up all his candies. If Cathy and Roy ate 4 candies and 8 candies respectively each day, Cathy would still have 200 candies when Roy had eaten up all his candies. How many candies did each of them have?

- 24. A 2-digit number is called reflexive if it is 27 more than the number obtained by reversing its digits. How many 2-digit numbers, which do not have 0 as either of their digits, are reflexive?
- 25. How many 3-digit numbers are odd, exactly divisible by 3, and less than 456?
- 26. A 2-digit number is called fascinating if the number obtained by reversing its digits is 75% larger than the original number. For example, 24 is fascinating because 42 is 75% larger than 24. How many two-digit numbers are fascinating?
- 27. A chess board has 32 black squares and 32 white ones. In how many ways can you choose one white and one black square on a chess board so that they are neither in the same row nor in the same column?
- 28. In the rectangle ABCD, M and N are the midpoints of AD and BC, respectively, and AC intersects MB and DN at P and Q, respectively. If AD is 24 cm and AB is 18 cm, what is the area of MPQD in cm2?
- 29. A container has a square base with a side of 8 cm. Twelve 4-cm cubes are placed inside in the container. Water is then poured into the container until it is  $\frac{5}{6}$  full. When all cubes are removed, the water level dropped to  $\frac{2}{3}$  the height of the container. Find the height of the container.
- 30. Eleanor was travelling from City F to City G at a uniform speed. She drove past Rachel who was travelling at a uniform speed of 84 km/h in the opposite direction. One and a half hours later, Eleanor reached City G while Rachel was still 39 km away from City F. If Eleanor took 4 hours to travel from City F to City G, what was the distance between the two cities?